

Siemens Competition

Math : Science : Technology

Regional Finalist

Names: Shakthi Shrima, Jacob Gurev, and Adam Forsyth

High School: Homeschool, Mira Loma High School, and Georgetown Day School, respectively

Mentor: Raffael Singer

Project Title: *Metacommutation of the Hurwitz Integers and the Projective Line over F_p* (Mathematics)

We completely characterize the metacommutation mapping for pairs of primes in the ring of Hurwitz quaternions as a group action isomorphic to the standard action of the projective general linear group over a finite field of order p on the projective line over a field of order p . In 2013, Cohn and Kumar investigated this mapping, calculating the sign and number of fixed points of the permutation of Hurwitz primes of norm p induced by a Hurwitz prime of norm q under metacommutation. Using our characterization, we then present simple proofs of the results in Cohn and Kumar's paper, the most notable of which is that the sign of the metacommutation map of a prime of norm q on the primes of norm p is the Legendre symbol of q on p . We also prove that all of the cycles which are not fixed points in a metacommutation map have the same length. Finally, we generalize the notion of metacommutation to rings with a division algorithm for primes of norm p , and show that our characterization continues to hold for any such ring.