

Siemens Competition

Math : Science : Technology

Regional Finalist

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Project Title: *Special Configurations of Triangle Centers* (Computer Science)

For a given triangle, the *circumcenter* is the unique point in the plane of the triangle at an equal distance from all three vertices. Similarly, the *orthocenter* is the point of intersection of the altitudes of the triangle. Euler proved that for any triangle, the midpoints of the sides, the feet of the altitudes and the midpoints of the segments joining the vertices of the triangle to the orthocenter lie on a circle. The center of this circle is the *9-point center* of the triangle.

There are infinite ways to define the *center* of a triangle. For any four points in the plane, the centers of the four resulting triangles determine a quadrilateral. We investigate under what conditions this quadrilateral satisfies some special property.

Among other things, we prove that, for any four points in a plane, of which three are noncollinear and all are not on a circle, the four-point configurations determined by the circumcenters and by the 9-point centers, respectively, are similar. Moreover, the similarity ratio is not constant but depends on the initial points in a way that is made explicit.