Siemens Competition
Math : Science : Technology
Regional Finalist

Names: Jonathan Chan and Michael Seaman
High School: Bergen County Academies, Homeschool
Mentor: Professor Keith Conrad
Project Title: *On the Distribution of Discriminants over a Finite Field*

A quadratic polynomial $ax^2 + bx + c$ has a discriminant, which is the familiar number $b^2 - 4ac$ from the quadratic formula. Any polynomial of degree higher than 2 also has a discriminant, which is a number expressible from the coefficients that describes important properties of the polynomial. Discriminants arise in many areas of mathematics, such as Galois theory, algebraic number theory, and elliptic curves.

Our research is concerned with discriminants of polynomials having coefficients in F_q, the finite field with q elements. It has been known for a long time, by a classical theorem of Stickelberger and Swan, that when q is odd, the monic polynomials with coefficients in F_q and nonzero discriminant have their discriminants equally distributed among the nonzero squares and non-squares in F_q. Recently, it was shown that as the degree tends to infinity, the discriminants of monic polynomials with common degree and coefficients in F_q are roughly equally distributed over F_q. Our paper focuses on an aspect of these ideas that has not been addressed before: uniform distribution of discriminants in specific degrees, rather than in the limit of large degrees. For an odd prime power q, we prove a sufficient condition for the equal distribution of discriminants of monic polynomials in $F_q[x]$ with a given degree, and show that this condition occurs infinitely often. We also prove an analogous result for even q. In addition, we show for each degree m greater than or equal to the characteristic of F_q that every number in F_q is the discriminant of a monic polynomial in $F_q[x]$ of degree m.