

# Siemens Competition

## Math : Science : Technology

### Regional Finalist

**Name:** Joseph Zurier

**High School:** Classical High School

**Mentor:** Ben Yang

**Project Title:** *Generalizations of the Joints Problem*

We explore generalizations of the joints problem introduced by B. Chazelle et al. A joint is formed when three noncoplanar lines intersect in  $\mathbf{R}^3$ , and other authors have proved an  $O(n^{3/2})$  bound on the number of joints formed by  $n$  lines. We narrow the constant in this bound to between  $(2^{1/2})/3$  and  $4/3$ , and explore the problem when the dimension of the space, the dimension of the intersecting hyperplanes, and the dimension of their mutual intersection is changed. We also consider cases where the intersecting hyperplanes do not all have the same dimension, focusing on the simplest nontrivial case in  $\mathbf{R}^4$ . This case is used to reconsider the original joints problem with an eye towards extending current results to higher dimensions. We prove an analogue of the joints theorem for this case and use it to give a new proof of the theorem in  $\mathbf{R}^3$ .