Siemens Competition
Math : Science : Technology
Regional Finalist

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Project Title: *Extremal Functions of Forbidden Multidimensional Matrices* (Computer Science)

Pattern avoidance is a central topic in graph theory and combinatorics. Pattern avoidance in matrices has applications in computer science and engineering, such as robot motion planning and VLSI circuit design. A d-dimensional zero-one matrix A avoids another d-dimensional zero-one matrix P if no submatrix of A can be transformed to P by changing some ones to zeros. A fundamental problem is to study the maximum number of nonzero entries in a d-dimensional $n \times \cdots \times n$ matrix that avoids P. This maximum number, denoted by $f(n, P, d)$, is called the extremal function.

Firstly, we establish the tight bound $\Theta\left(n^{d-1}\right)$ on $f(n, P, d)$ for every d-dimensional tuple permutation matrix P. This tight bound has the lowest possible order that an extremal function of a nontrivial matrix can ever achieve. Secondly, we show that $f(n, P, d)$ is super-homogeneous for a class of matrices P. We use this super-homogeneity to show that the limit inferior of the sequence $\left\{\frac{f(n,P,d)}{n^{\{d-1\}}}\right\}$ has a lower bound $2^{\Omega\left(k^{(d-1)/d^2}\right)}$ for a family of $k \times \cdots \times k$ permutation matrices P. We also improve the upper bound on the limit superior from $2^{O(k \log k)}$ to $2^{O(k)}$ for all $k \times \cdots \times k$ permutation matrices and show that the new upper bound also holds for tuple permutation matrices.