

Siemens Competition

Math : Science : Technology

Regional Finalist

Names: Noah Golowich

High School: Lexington High School

Mentor: László Miklós Lovász

Project Title: *Resolving a Conjecture on Degree of Regularity, with some Novel Structural Results (Computer Science)*

Determining the degree of regularity of linear equations is an important problem in the area of Ramsey theory on the integers. This topic has potential applications in other areas of mathematics and in computer science. A linear equation is *r-regular*, if, for every *r*-coloring of the positive integers, there exist positive integers of the same color which satisfy the equation. The *degree of regularity* of an equation is the largest *r* such that the equation is *r*-regular. The problem of determining the degree of regularity of an arbitrary linear equation is very difficult and largely open. In 2005, as an approach to this problem, Fox and Radoičić conjectured that for any $n \geq 2$, the equation $x_1 + 2x_2 + \dots + 2^{n-2}x_{n-1} - 2^{n-1}x_n = 0$ has a degree of regularity of $n - 1$. Fox and Radoičić's conjecture has remained open since 2005; in this project, we resolve this conjecture in its entirety. Moreover, we prove a broader condition for *r*-regularity of a linear homogeneous equation, generalizing our proof of Fox and Radoičić's conjecture. Our method of proof is flexible and allows us to prove a number of additional novel extensions which are general structural results in this field.