

“On the Solution of the Dirichlet Problem with Rational Boundary Data”**Michael Viscardi, Josan Academy, San Diego, CA**

Abstract: We study the solution of the Dirichlet problem in a simply connected bounded domain Ω in $\mathbb{R}^2 \cong \mathbb{C}$ given rational boundary data of one complex variable. We obtain and prove several new results, including the following:

1. The solution to every Dirichlet problem with rational boundary data of one complex variable is rational if and only if a Riemann map $f: \Omega \rightarrow D$ is rational.
2. The solutions to the above Dirichlet problems are rational if and only if the solution is rational for a single, relatively ‘simple’ data function, namely, the function $1/(z - a)$, where a is a point in the domain.
3. The Bergman kernel $K(z, w)$ is rational (as a function of z and w) if and only if $K(z, a_1)$ and $K(z, a_2)$ are rational functions of z , where a_1 and a_2 are any two distinct points in the domain. In particular, this implies that a Riemann map $f: \Omega \rightarrow D$ is rational if and only if $K(z, a_1)$ and $K(z, a_2)$ are rational functions of z .

“Closure Properties of D_{2p} in Finite Groups”**Po-Ling Loh, James Madison Memorial High School, Madison, WI -- 2004-05 National Individual Finalist**

Abstract: We discuss the generalized version of a question posed by the topologist E. Farjoun about closed embeddings of a finite group H in a finite group G , where H is closed in G if every element of $\text{Hom}(H, G)$ extends to a unique element of $\text{End}(G)$. We study the case $H = D_{2p}$, for p an odd prime, and show that if $H = D_{2p}$ is closed in G and G is solvable, then $H = G$. We further conjecture that the solvability of G is a necessary condition for $H = G$.

“Monte Carlo Simulations of the Hill Dose Response Model”**Nicole Pranke, Perpich Center for Arts Education, Golden Valley, MN and Ashley Fry, Half Hollow Hills High School East, Dix Hills, NY -- 2004-05 National Team Finalists**

Abstract: The Monte Carlo method is a statistical tool incorporated into a variety of fields, ranging from economics to biology. Since its development as part of the Manhattan Project during World War II, Monte Carlo simulations have been used to illustrate real life error-containing scenarios. In our experiment, we performed Monte Carlo simulations for variations of the Hill Dose Response Model, a logistic formula used frequently to model the pharmacodynamic effect of drugs. In addition, we tested four weighting schemes, in which we weighted the data with the predicted, observed, or ideal y values, or simply did not weight at all. We also examined the effect of alterations in the x -data point design, number of replications of the design, and steepness of the Hill Model. These factors affect the accuracy and precision of the four parameters of the Hill Model: E_{max} , D_{50} , m , and B . Our purpose is to determine which combination of specific factors results in producing parameter values (1) that are closest to our original ideal parameters, smallest bias, and (2) that have the smallest variance, greatest precision. We believe our approach will assist the successful development of new chemotherapeutic drug combinations against cancer and other diseases.

“Investigations of the Number Derivative”

Linda Westrick, Maggie L. Walker Governor’s School for Government and International Studies, Richmond, VA – 2003-04 National Individual Finalist

Abstract: This paper explores the number derivative n' , a function defined in terms of the prime factorization of a positive integer n . We find an explicit formula and bounds for the function and investigate how the prime factorizations of n and n' are related. We then extend the function (originally defined only on $\mathbb{Z}^+ \cup \{0\}$) to the rational numbers and find all x in \mathbb{Q}^+ such that $x' = 0$. We prove that the number derivative is discontinuous over all \mathbb{Q}^+ . Finally, we analyze the equation $x' = ax$ for x in \mathbb{Q}^+ and x in \mathbb{Z}^+ , and find the conditions under which this equation has a solution.

Mentor: Mr. Pavlo Pylyavskyy

“Eccentric Graphs of Block Graphs and Trees”

Hannah Chung, Lyndon Baines Johnson High School, Austin, TX, Araceli Fernandez, Harlandale High School, San Antonio, Yidou “David” Wang, Lincoln High School, Portland, OR - 2002-03 National Team Finalists

Abstract: The eccentricity $e(v)$ of a vertex v in a connected graph G is the maximum distance from v to any other vertex in G . A vertex u is an eccentric vertex of v if $d(u, v) = e(v)$. Let $EC(G)$ denote the eccentric graph of G , defined as a graph having the vertex set of G , with two vertices u and v in $EC(G)$ adjacent if and only if $dG(u, v) = eG(u)$ or $dG(u, v) = eG(v)$. We examine the properties of the eccentric graphs of trees and blocks. In particular, we prove that for any tree block G , $EC(G)$ is connected. We apply this result to all trees. For T a tree, we prove that $rad(EC(T)) = 2$ and $diam(EC(T)) = 3$. In addition, we prove results relating the centers and peripherals of T to the centers and peripherals of $EC(T)$.

Mentor: Mrs. Diana Gu

“Poset-Game Periodicity”

Steven J. Byrnes, Roxbury Latin School, Lexington, MA – 2002-03 National Individual Winner

Abstract: In this paper, we explore poset games, a large class of combinatorial games which includes Nim, Chomp, Hackendot, Subset-Takeaway, and others. We prove a general theorem about poset games, which we call the Poset Game Periodicity Theorem: as a poset expands along two chains, losing positions and positions with any fixed g -value have a periodic pattern.

We use the theorem to (1) find polynomial-time winning strategies for a new, large class of poset games, (2) resolve two open conjectures about the game of Chomp, and (3) prove several important results about the computational complexity of calculating g -values in poset games.

Mentor: Mr. Edward Early

“Rainbow Ramsey Theory: Rainbow Arithmetic Progressions and Anti-Ramsey Results”

Jacob Fox, William Hall High School, West Hartford, CT – 2001-02 National Individual Finalist

Abstract: The theorems we prove are the first results in literature regarding rainbow (or anti chromatic) progressions. We say a progression is *rainbow* if all of its terms are different colors. The conjectures we state and theorems we prove can be thought of as the first counterparts of classical theorems in Ramsey theory, such as van der Waerden’s, Rado’s and Szemerédi’s theorems [9].

In this paper, we prove a stronger statement than a conjecture by Nešetřil et al. If $X \subset \mathbb{N}$, let $X(n) = |\{x \mid x \in X \text{ and } x \leq n\}|$. We prove that if \mathbb{N} is partitioned into three classes, R , G , and B , whose elements respectively are denoted as red, green and blue, such that $\limsup_{n \rightarrow \infty} (\min(R(n), B(n), G(n)) - n/6) = +\infty$, then there exists a 3-term arithmetic progression. We also prove in this paper that if \mathbb{Z}_n is partitioned into three classes, R , G , and B , such that the $\min(|R|, |B|, |G|) > n/q$, where q is the smallest prime factor of n , then there exists a rainbow 3-term arithmetic progression.

“Surreal Dimensions and their Application”

Alexandra Ovetsky, Central High School, Philadelphia, PA -- 2001-02 National Individual Finalist

Abstract: In this paper we introduce the concept of surreal dimensions as a refinement of fractal dimensions. We show that surreal dimensions distinguish many more irregularities in scaling than do real valued dimensions. We also show how surreal dimensions can be applied to the study of the sets of uniqueness in Fourier analysis.

“A Proof of the Collatz Conjecture for Rational Patterns”

Peter Behroozi, Malcom Price Laboratory School, Cedar Falls, Iowa – 2001-02 National Individual Finalist

Abstract: The Collatz Conjecture is a famous problem in number theory that has connections to many other fields of mathematics. Due to its inherent difficulty, it has remained unsolved since its first appearance in the 1930's. Although this paper does not give proof of the Conjecture, it provides several important results that could make the proof a much simpler task.

First, a formula is derived to obtain all numbers whose trajectories under the Collatz Function begin with a given number of odd integers. Next, a formula is derived to find numbers whose trajectories begin with an odd integer followed by a given number of even integers. Finally, it is proven that every positive integer has a unique trajectory and that certain trajectories do not exist.

“The Generalization of the deBruijn Edge Sums”

Cynthia Chi, William P. Clements High School, Sugar Land, Texas; Charles Hallford, Texas Academy of Mathematic and Science, Denton, Texas; Rebecca Williams, North Lamar High School, Paris, Texas -- 2001-02 National Team Finalists

Abstract: Our object is to characterize the edge sums for all deBruijn interconnection networks. An interconnection network is a highly symmetrical connected graph of order n nodes, size m edges, connectivity κ and diameter d , where n and κ are large but m and d are small.

Many interconnection networks are defined algebraically in such a way that each node has an integer value. Then every edge can be assigned the sum of the two nodes it joins. These numbers are called the edge sums of the graph. Graham and Haray defined the edge sum problem of any particular graph to be the characterization of the set of edge sums for that graph. These two, also, presented the solution for hypercubes.